BRINGING SANITY TO MARKETABILITY DISCOUNTS: A NEW DATA SOURCE

By Espen Robak and Lance S. Hall

The “benchmark approach” has had its day in court and has been found wanting. Other methods have proven unworthy alternatives for determining marketability discounts. What is needed now, more than ever, is hard evidence – not just of what an “accepted” or “average” discount might be, but also of how the discount of a particular investment should differ from the average. Will the FMV Restricted Stock Study be the solution? The authors here provide a preview of the FMV Study’s main conclusions and benefits.

INTRODUCTION

In the fall of 1998, a huge hedge fund run by some of the smartest people on Wall Street collapsed – and almost pulled down the U.S. financial markets with it. Among the many reasons for the losses of the fund was a peculiar bet: a bet against the value of liquidity.

Long Term Capital Management had gambled on the yield differential between the 30-year and 29-year Treasury bonds. The fund had taken enormous long positions in the 29-year bond and offsetsing short positions in the 30-year bond, in order to exploit the approximately 5-basis point yield differential in favor of the 29-year bond. The 29-year bond is almost always priced below the 30-year bond (i.e., at a higher yield) because it is a less liquid security. The LTCM fund managers were betting that this discount for lack of liquidity would drop. How wrong they were! When Russia defaulted on its obligations, investors changed their view of the riskiness of the bond markets. Did they like liquidity? They did, very much. The yield spread soared, to 35 basis points. With this and other losses, the investment capital of LTCM evaporated. An expensive bailout was required to calm investors and avoid a market meltdown.

While the LTCM debacle was a real world lesson in the value of liquidity, the issue of liquidity arises most often in Tax Court. If a tax is owed on the value of an equity interest in a privately held entity, where the interest lacks the liquidity of an active market, the taxpayer will benefit by taking a large discount for the illiquidity of that interest. The IRS, of course, benefits from taking a smaller discount. The many Tax Court cases on point provide insight into the methods used to support marketability discounts. The method most often used is the restricted stock analysis. Unfortunately, the benchmark

---

1 Espen Robak, CFA, is an Assistant Vice President at FMV Opinions, Inc., in San Francisco, California. Lance S. Hall is a Managing Director and co-founder of FMV and can be reached at the firm’s Orange County, California, office. Significant research help for this article was provided by Luis J. Mendoza at FMV’s Orange County office.
studies used until now have not provided the analyst with sufficient detail in order to differentiate between discounts for different entities. The FMV Restricted Stock Study, soon to be released to the appraisal community, our clients, the IRS, and the courts, provides rich detail on more than 230 restricted stock transactions.

**Lack-of-Marketability Approaches Used in Tax Court**

*Restricted Stock Analyses*

Restricted stock studies are superior to other methods because they compare directly the price paid for restricted stock sold by a firm in a private placement (illiquid stock, see sidebar 1) with the contemporaneous market price for the otherwise identical stock traded on an exchange or over-the-counter system. The following table summarizes the results of previously published restricted stock studies:2

<table>
<thead>
<tr>
<th>Restricted Stock Study</th>
<th>Years Covered</th>
<th>Average Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Institutional Investor Study</td>
<td>1966-1969</td>
<td>25.8%</td>
</tr>
<tr>
<td>Gelman</td>
<td>1968-70</td>
<td>33.0%</td>
</tr>
<tr>
<td>Trout</td>
<td>1968-72</td>
<td>33.5%</td>
</tr>
<tr>
<td>Moroney</td>
<td>Not Specified</td>
<td>35.6%</td>
</tr>
<tr>
<td>Maher</td>
<td>1969-73</td>
<td>35.4%</td>
</tr>
<tr>
<td>Standard Research Consultants</td>
<td>1978-82</td>
<td>45.0%</td>
</tr>
<tr>
<td>Willamette Management Associates</td>
<td>1981-84</td>
<td>31.2%</td>
</tr>
<tr>
<td>Silber</td>
<td>1981-88</td>
<td>33.8%</td>
</tr>
<tr>
<td>Management Planning</td>
<td>1980-96</td>
<td>27.1%</td>
</tr>
</tbody>
</table>

The studies listed above were mostly performed on small samples of restricted stock transactions. Also, most of the studies listed in the table provide little or no detail about how the discount varies around the average. However, the most comprehensive of these studies, the Institutional Investor Study,3 does provide some detail: the discounts in this study tended to be higher for transactions involving companies with lower revenues and net income, and for companies traded in the over-the-counter market. Common for all the restricted stock studies, however, is the fact that since the underlying data has never been made public, it is impossible to verify any of their conclusions. Moreover, the average discounts in most of the older studies are contradicted by studies published in academic journals.4 Furthermore, even if these studies were not contradicted by more rigorous academic studies, the use of an average provides little insight into what an appropriate

---


discount should be for a specific interest. In other words, it begs the question, “Is the subject asset better or worse than the average?” Without the underlying data, it is literally impossible to know.

While the restricted stock approach to determining discounts for lack of marketability based upon the above studies has its drawbacks, this approach is the most widely used and most often accepted by Tax Court.

**Pre-IPO Studies**

The leading alternative to restricted stock studies proposed for liquidity discount determinations are pre-IPO studies. Currently, two different series of pre-IPO studies performed by two competing firms are available. The two studies each in their own way illustrate the main drawbacks of using the pricing of pre-IPO transactions in common stock as a proxy for the value of liquidity. To put it as briefly as possible, many of the transactions in the studies either take place too far from the initial public offering date or are not arm’s-length transactions. The problem with transactions taking place too far prior to the IPO is that the typical pre-IPO firm goes through a radical transformation, in terms of products and services offered, customers served, depth and breadth of management, and other factors. Furthermore, the pre-IPO studies have a successful IPO bias that drives the presumed average discount up

---

significantly. These significant shortcomings cast doubt on the relevance of the pre-IPO studies for determining lack of marketability discounts for private equity.

Why Liquidity is Not a Put Option

It has been suggested that the discount for liquidity can be estimated by determining the price of a put option, the idea being that the owner of an illiquid security can simply purchase a put option for the security, with a time to expiration equal to the estimated duration of the security’s lack of liquidity. This, then, will provide liquidity and insure the owner against any future losses on his security.

The problem with this method is that the standard option pricing methodologies available provide no insight into the value of liquidity. Indeed, one of the assumptions behind the Black-Scholes model, the most widely used valuation model for options, is that the security can be continuously traded. When valuing a put option on a security with limited marketability, the most appropriate method is either to discount the underlying security for lack of liquidity (and then apply the Black-Scholes model with the adjusted input data), or to apply a marketability discount directly to the option value indication from the Black-Scholes formula. In fact, institutions active in the “market” for private warrants purchase them at significant discounts to their calculated Black-Scholes values because of their illiquidity.

A put option has value in excess of its intrinsic “exercise” value because of its asymmetrical payoff:

**Sidebar 2: History of Restricted Stock.** Before the adoption of Rule 144, the SEC and the courts were required to inquire into the subjective intent of the purchaser at the time of her purchase, to make sure she did not purchase the stock with a view to distribution, before approving any resale. Otherwise, the registration requirements of the Act could have been avoided through a two-step public distribution: a private sale from the company to an investor, exempt under section 4(2), followed by an immediate resale to the public, exempt under section 4(1). This subjective test evolved in the courts into a complicated set of rules, focusing on how long the purchaser had held the securities and whether the purchaser had undergone a change of circumstances that might force a sale.

**Rule 144.** In January 1972, the SEC adopted Rule 144 as an objective safe harbor for the resale of restricted securities. To prevent a seller from purchasing unregistered securities with a view to resale, Rule 144 required an initial holding period of 2 years, followed by a dribble-out period (see sidebar 1).

**Rule 144 (k).** In 1983, Rule 144 (k) was added in an amendment to the Rule. This rule allowed any person or entity that is not an affiliate of the issuer to sell all remaining securities, without volume limits, after a holding period of at least three years.

**1990 Amendment.** The SEC amended Rule 144 again in 1990. The amendment allowed the purchaser of restricted stock to add to his own holding period the length of time his securities had been held by any previous owners, as long as the previous owners were not affiliates of the issuer.

**1997 Amendment.** In 1997, Rule 144 was again amended to improve the marketability of restricted stock. This time, the rule was amended to shorten the initial holding period from two years to one year and the ultimate 144(k) holding period for non-affiliates from three years to two years.
when the option is at-the-money (the underlying security’s value equals the option’s exercise price), the option holder gets the benefit of any future value appreciation, but suffers no losses from future value reductions.6

The QMDM

Recently, the Quantitative Marketability Discount Model has been introduced as an alternative method for determining a marketability discount, especially for the shares of privately held firms.7 The model estimates the marketability discount from a range of inputs across three main parameters: the growth rate of the value of the security, the required rate of return on that security, and the time to liquidity for the (illiquid) security. The model is based on the basic discounted cash flow method, slightly modified. Unfortunately, the model is a tautological exercise (see sidebar 6) that requires the foreknowledge of the discount for lack of marketability in order to derive a premium return to compensate an investor for assuming an illiquid investment, in order to determine a lack of marketability discount. Furthermore, the model substitutes the bias of the user for empirical evidence, to support any conclusion its user may want. On this basis, it becomes easier to understand the lack of enthusiasm of judges for valuation reports where the QMDM has been applied.8

SIDEBAR 3: Liquidating Restricted Stock Positions

The owner of restricted stock that wishes to liquidate his position prior to the end of the holding period has several options for doing so, including:

1. Persuading the issuer to register the stock. The restricted stock purchaser may have obtained this option when negotiating the securities purchase agreement(s) for the private placement (see sidebar 4).
2. Selling the position to an institutional investor that specializes in purchasing positions in restricted stock at a discount.
3. A hedging transaction where an institution will first write a “collar” on the stock to remove most of the economic risk of the restricted stock position, then lend aggressively against the hedged position. This transaction has a cost to the investor, similar to the discount taken in a direct sale.

Both the hedging and sales transactions described above are practicable mostly for highly liquid stocks, where publicly traded options are available and/or short selling is possible. Such transactions are rarely possible for the stock of smaller companies.

Which Approach is Best, and How Can its Weaknesses be Overcome?

The Tax Court has – over decades – favored the use of restricted stock studies. However, as evidenced by several recent cases, the court is also unsatisfied with the benchmark averages and the lack of detail provided by the benchmark studies. In Kosman, the Tax

---

6 For a more thorough discussion of the problems with using option-pricing methods for “pricing” liquidity, please see Black-Scholes and Marketability: Another View, David Eckstein, Valuation Strategies, January/February 1999.
7 See Quantifying Marketability Discounts: Developing and Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests, Z. Christopher Mercer, Peabody Publishing.
Court had a chance to review certain of the conclusions of the FMV Study.\(^9\) Because the FMV Study is larger, more detailed, and more comprehensive in terms of the comparative variables than the other studies relied upon, the appraiser using the FMV database won an easy victory on the marketability discount. However, the full detail of the FMV Study has not been made available to the general appraisal community – until now. We confidently predict that this new data-source will greatly improve matters over both the “benchmark” approach and any other alternative model.

### The FMV Restricted Stock Study

The FMV Study includes 230 transactions from 1980 to April 1997, the date of the most recent amendment of Rule 144 (see sidebar 2). A review of the construction of the study is beyond the scope of this article; however, the methodology is available with the data.

**SIDEBAR 4: Registration Rights Agreements.** Even though the resale limitations under Rule 144 have been reduced somewhat over the years, investors still often look for the issuer in a private placement transaction to provide them with a possible liquidity event prior to the end of the holding period. Consequently, investors often require, and issuers often grant, some form of a registration rights agreement to be signed, either as part of the stock purchase agreement or as a separate side agreement thereto. These registration rights provisions typically either (1) require the company to register the shares or a portion of the shares within some specified time-frame, at either the company’s or the investor’s expense (demand registrations) or (2) require the company to include the shares or a portion of the shares in any future public offerings that the company makes, if any (piggyback registrations).

Clearly, the presence of a registration rights agreement tends to improve the liquidity of restricted stock. However, there is no “standard” registration rights agreement and it is difficult to gauge the precise impact that a particular registration rights agreement will have on the marketability of a stock issue, or on the discount for lack of marketability. It is safe to say, however, that any block of restricted stock, with or without registration rights, is less liquid than fully registered, unrestricted shares of the same issuer.

**Overall Sample Description**

The overall average discount for all 230 transactions in the FMV Restricted Stock Study is 22.3 percent and the median discount is 20.1 percent. The standard deviation of the sample is 17.2 percent. The sample distribution is shown in the histogram below:

As shown in the figure, the discounts vary significantly throughout the sample. Clearly, with the significant difference in the discount for different firms, relying on “benchmark” averages is inappropriate.

With the detail contained within the FMV Study, a comparative analysis with the subject entity can be performed across many variables. Making comparisons by industry, by trading market, by total assets, and by market-to-book ratio, are only a few of the many ways the data can be broken down.

**Differences by Industry.** In the table below we investigate whether or not there are significant differences between firms placing restricted stock privately based on industry (4-digit SIC).

<table>
<thead>
<tr>
<th>SIC Range</th>
<th>1000s</th>
<th>2000s</th>
<th>3000s</th>
<th>4000s</th>
<th>5000s</th>
<th>6000s</th>
<th>7000s</th>
<th>8000s</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Transactions</td>
<td>14</td>
<td>31</td>
<td>72</td>
<td>21</td>
<td>16</td>
<td>24</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>Average Discount</td>
<td>20.3%</td>
<td>23.3%</td>
<td>24.9%</td>
<td>23.4%</td>
<td>12.0%</td>
<td>12.9%</td>
<td>26.8%</td>
<td>23.4%</td>
</tr>
<tr>
<td>Median Discount</td>
<td>15.9%</td>
<td>18.4%</td>
<td>24.9%</td>
<td>14.4%</td>
<td>12.5%</td>
<td>11.5%</td>
<td>23.7%</td>
<td>25.6%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>15.9%</td>
<td>15.1%</td>
<td>17.8%</td>
<td>18.8%</td>
<td>7.6%</td>
<td>19.6%</td>
<td>17.5%</td>
<td>15.3%</td>
</tr>
</tbody>
</table>

Interestingly, while there is a significantly higher number of transactions in the SIC codes in the 3000-3999 range, the average and median discounts do not appear to vary significantly across industries, with the exception of the 5000-6999 range, where both the average and median discounts are lower than the average. For the 5000-5999 range (distributors and retailers) the sample is quite small, which may skew the results. The lower-than-average discount for the larger sample in the 6000-6999 range can perhaps be explained by the many banks and financial institutions in this industry group.
Trading Market. A comparison between exchange-traded and over-the-counter traded equities shows significantly higher discounts for over-the-counter traded securities. The median discount for exchange-traded securities is 15.3 percent, while the median discount for over-the-counter traded securities is 23.2 percent.

Percentage Block Size. The FMV Restricted Stock Study also provides conclusions particularly important to the valuation of privately held firms. Since privately held firms have no market for their stock (and many smaller, less attractive, firms have no realistic prospect ever of establishing a market), the discounts for restricted stock with longer-than-average holding periods (as indicated by Rule 144 dribble-out provisions, see sidebar 1) are particularly applicable. The FMV Study confirms that for the largest, least-liquid blocks, the average and median discount can be as much as 10-15 percentage points higher than the average and median discounts for small block sizes.

Other Factors. The impact of risk on the marketability discount is significant. Smaller, less profitable entities, and those with a higher degree of balance sheet risk, will tend to have higher discounts. The following table provides a firm characteristic comparison between high-discount transactions and low-discount transactions. The sample is divided into quintiles, based on the distribution of the restricted stock discount, and medians are computed for each group across all parameters.

<table>
<thead>
<tr>
<th>Quintile</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage Discount</td>
<td>2.9%</td>
<td>12.6%</td>
<td>21.1%</td>
<td>31.2%</td>
<td>43.7%</td>
</tr>
<tr>
<td>Market-to-Book Ratio</td>
<td>3.97</td>
<td>5.05</td>
<td>4.88</td>
<td>8.12</td>
<td>7.92</td>
</tr>
<tr>
<td>Total Assets ($000s)</td>
<td>43,585</td>
<td>31,404</td>
<td>16,305</td>
<td>10,725</td>
<td>5,994</td>
</tr>
<tr>
<td>Price per Share</td>
<td>$10.19</td>
<td>$8.50</td>
<td>$6.00</td>
<td>$3.42</td>
<td>$3.75</td>
</tr>
</tbody>
</table>

As shown in the table above, assets and price per share tend to decrease and the market-to-book ratio tends to increase as the discount increases. Higher risk, which is associated with smaller firm size and higher market-to-book ratios, tends to increase the discount. The discount also increases significantly with decreasing stock prices. Price per share is an indicator of size and firm risk. It can also be an indicator of liquidity. If the stock price drops below certain thresholds ($5 for NYSE stocks, $1 for Nasdaq stocks), the stock is in danger of being delisted. The average discount in the FMV Restricted Stock Study for all private placements with a per-share price below $5 is 29.0 percent and the median discount is 30.4 percent.
Other inferences drawn from the FMV Restricted Stock Study (including revenue, income, dividend payments, dollar block size, book value, market value, and trading volume) also confirm the relationship between risk and the lack of marketability discount. The following are some additional conclusions revealed by the FMV Study:

- **Highly profitable firms** tend to have lower discounts than the average. The top decile of the sample, arranged by profitability, has a median discount of 11.0 percent.
- **Dividend-paying firms** tend to have lower discounts than the average.
- The amount of the private placement shows a strong negative correlation with the discount.
- **Private placements** where the stock has been issued with registration rights (see sidebar 4) have lower discounts than private placements without such rights.
- **Firms with low shareholders’ equity** tend to have significantly higher discounts than the average. The bottom quintile of the sample, arranged by book value, has an average discount of 29.0 percent.
- **Stocks that are more liquid** (high trading volume) than the average tend to have lower discounts. Private

**SIDEBAR 6: The QMDM: Circular Reasoning?**

Under the QMDM, the marketable-equivalent value of the investment as of the valuation date is projected to the end of the investment horizon with the chosen growth rate, then discounted back to the present with the required rate of return for the illiquid investment. Ignoring dividends, the formula is as follows:

\[
P_V = V_m \frac{(1 + g)^T}{(1 + r_{nm})^T}, \text{ where:}
\]

- \(P_V\) = the value of the illiquid investment,
- \(V_m\) = the value of a fully liquid investment,
- \(T\) = the time until the illiquid investment becomes liquid,
- \(g\) = the growth rate of the investment from the “base” fully liquid value, and
- \(r_{nm}\) = the required rate of return on the illiquid investment.

The growth rate is simply the required rate of return for a fully liquid investment \((r_m)\). (If that were not the case, \(P_V\) for a liquid investment would not equal \(V_m\).) Renaming the growth rate, the formula is restated:

\[
P_V = V_m \frac{(1 + r_m)^T}{(1 + r_{nm})^T}
\]

The analyst has likely already determined \(r_m\) as part of the process of determining \(V_m\). The two remaining steps, then, are estimating the incremental required rate of return for lack of liquidity \((r_{nm} - r_m)\) and selecting the time horizon.

Clearly, if a well-supported discount is the goal, the incremental required rate of return for lack of liquidity is best estimated using data from restricted stock deals. Applying the model to known restricted stock transactions where both \(V_0\) and \(P_V\) are known, and the time horizon can be estimated with some degree of accuracy, and making reasonable assumptions about \(r_{nm}\) yields estimates of \(r_{nm}\). So, the QMDM starts with discounts on restricted stock to determine \((r_{nm} - r_m)\), then uses this incremental rate or return to determine the discount.
placements that are very large compared with the monthly trading volume for the stock will tend to sell at higher discounts.

**SUMMARY AND CONCLUSIONS**

While many methods have been used to try to ascertain the impact of lack of marketability on the value of a privately held investment, the Tax Court has tended to rely on restricted stock analyses. However, the courts are clearly displeased with the results from using the prevalent restricted stock studies. The “benchmark” analysis based on average discounts and little else is unlikely to prevail in the future. As the average does not answer the question, “Is the subject asset better or worse than the average?,” these studies will likely fail when compared against a restricted stock study with sufficient data to allow for an in-depth analysis of the factors that impact the magnitude of the discount.